

對稱的相乘式 / 資訊融入數學學習一例

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$$13 \times 93 = 39 \times 31 \quad 69 \times 32 = 23 \times 96$$

$13 \times 93 = 39 \times 31$ ， $69 \times 32 = 23 \times 96$ ，我們就稱左列的算式叫做二位數的對稱相乘式。如文後所列表一，表中各式是利用 Decimal BASIC 設計程式跑出來的，可以參閱我發表於昌爸工作坊的「介紹 Decimal BASIC 的程式設計」 <http://www.mathland.idv.tw/soft/decbasic.htm>。九年一貫實施後的中學生，被要求具備將情境中和數學相關的資料資訊化，並能將待解的問題轉化成數學問題的基本數學能力(參閱 89 版數學領域暫行綱要能力指標)。

學生只要具備 A-3-1:能用 x, y, \dots 表徵生活情境中的未知量與變量，而且完成 A-4-8:能使用乘法公式。應該可以列式找出二位數的對稱相乘式的規律。假設這四個二位數分別是 $10x+y, 10a+b, 10b+a, 10y+x$ ，其中 $9 \geq y, b \geq 0, 9 \geq x, a > 0$ 。因為 $(10x+y)(10a+b) = (10b+a)(10y+x)$ ，利用分配律展開並使用等量公理，可以得到 $ax = by$ 。

這問題可以往縱向延伸學習，討論三位數以上的對稱相乘式。也可以往橫向加廣學習，利用推理方式探討對稱相乘式的組數。

[附錄] 表一

1	$11 \times 11 = 11 \times 11$	26	$21 \times 36 = 63 \times 12$	51	$29 \times 92 = 29 \times 92$	76	$39 \times 31 = 13 \times 93$
2	$11 \times 22 = 22 \times 11$	27	$21 \times 48 = 84 \times 12$	52	$31 \times 13 = 31 \times 13$	77	$39 \times 62 = 26 \times 93$
3	$11 \times 33 = 33 \times 11$	28	$22 \times 11 = 11 \times 22$	53	$31 \times 26 = 62 \times 13$	78	$39 \times 93 = 39 \times 93$
4	$11 \times 44 = 44 \times 11$	29	$22 \times 22 = 22 \times 22$	54	$31 \times 39 = 93 \times 13$	79	$41 \times 14 = 41 \times 14$
5	$11 \times 55 = 55 \times 11$	30	$22 \times 33 = 33 \times 22$	55	$32 \times 23 = 32 \times 23$	80	$41 \times 28 = 82 \times 14$

6	$11 \times 66 = 66 \times 11$	31	$22 \times 44 = 44 \times 22$	56	$32 \times 46 = 64 \times 23$	81	$42 \times 12 = 21 \times 24$
7	$11 \times 77 = 77 \times 11$	32	$22 \times 55 = 55 \times 22$	57	$32 \times 69 = 96 \times 23$	82	$42 \times 24 = 42 \times 24$
8	$11 \times 88 = 88 \times 11$	33	$22 \times 66 = 66 \times 22$	58	$33 \times 11 = 11 \times 33$	83	$42 \times 36 = 63 \times 24$
9	$11 \times 99 = 99 \times 11$	34	$22 \times 77 = 77 \times 22$	59	$33 \times 22 = 22 \times 33$	84	$42 \times 48 = 84 \times 24$
10	$12 \times 21 = 12 \times 21$	35	$22 \times 88 = 88 \times 22$	60	$33 \times 33 = 33 \times 33$	85	$43 \times 34 = 43 \times 34$
11	$12 \times 42 = 24 \times 21$	36	$22 \times 99 = 99 \times 22$	61	$33 \times 44 = 44 \times 33$	86	$43 \times 68 = 86 \times 34$
12	$12 \times 63 = 36 \times 21$	37	$23 \times 32 = 23 \times 32$	62	$33 \times 55 = 55 \times 33$	87	$44 \times 11 = 11 \times 44$
13	$12 \times 84 = 48 \times 21$	38	$23 \times 64 = 46 \times 32$	63	$33 \times 66 = 66 \times 33$	88	$44 \times 22 = 22 \times 44$
14	$13 \times 31 = 13 \times 31$	39	$23 \times 96 = 69 \times 32$	64	$33 \times 77 = 77 \times 33$	89	$44 \times 33 = 33 \times 44$
15	$13 \times 62 = 26 \times 31$	40	$24 \times 21 = 12 \times 42$	65	$33 \times 88 = 88 \times 33$	90	$44 \times 44 = 44 \times 44$
16	$13 \times 93 = 39 \times 31$	41	$24 \times 42 = 24 \times 42$	66	$33 \times 99 = 99 \times 33$	91	$44 \times 55 = 55 \times 44$
17	$14 \times 41 = 14 \times 41$	42	$24 \times 63 = 36 \times 42$	67	$34 \times 43 = 34 \times 43$	92	$44 \times 66 = 66 \times 44$
18	$14 \times 82 = 28 \times 41$	43	$24 \times 84 = 48 \times 42$	68	$34 \times 86 = 68 \times 43$	93	$44 \times 77 = 77 \times 44$
19	$15 \times 51 = 15 \times 51$	44	$25 \times 52 = 25 \times 52$	69	$35 \times 53 = 35 \times 53$	94	$44 \times 88 = 88 \times 44$
20	$16 \times 61 = 16 \times 61$	45	$26 \times 31 = 13 \times 62$	70	$36 \times 21 = 12 \times 63$	95	$44 \times 99 = 99 \times 44$
21	$17 \times 71 = 17 \times 71$	46	$26 \times 62 = 26 \times 62$	71	$36 \times 42 = 24 \times 63$	96	$45 \times 54 = 45 \times 54$
22	$18 \times 81 = 18 \times 81$	47	$26 \times 93 = 39 \times 62$	72	$36 \times 63 = 36 \times 63$	97	$46 \times 32 = 23 \times 64$
23	$19 \times 91 = 19 \times 91$	48	$27 \times 72 = 27 \times 72$	73	$36 \times 84 = 48 \times 63$	98	$46 \times 64 = 46 \times 64$
24	$21 \times 12 = 21 \times 12$	49	$28 \times 41 = 14 \times 82$	74	$37 \times 73 = 37 \times 73$	99	$46 \times 96 = 69 \times 64$
25	$21 \times 24 = 42 \times 12$	50	$28 \times 82 = 28 \times 82$	75	$38 \times 83 = 38 \times 83$	100	$47 \times 74 = 47 \times 74$

101	$48 \times 21 = 12 \times 84$	126	$62 \times 39 = 93 \times 26$	151	$72 \times 27 = 72 \times 27$	176	$86 \times 34 = 43 \times 68$
102	$48 \times 42 = 24 \times 84$	127	$63 \times 12 = 21 \times 36$	152	$73 \times 37 = 73 \times 37$	177	$86 \times 68 = 86 \times 68$
103	$48 \times 63 = 36 \times 84$	128	$63 \times 24 = 42 \times 36$	153	$74 \times 47 = 74 \times 47$	178	$87 \times 78 = 87 \times 78$
104	$48 \times 84 = 48 \times 84$	129	$63 \times 36 = 63 \times 36$	154	$75 \times 57 = 75 \times 57$	179	$88 \times 11 = 11 \times 88$
105	$49 \times 94 = 49 \times 94$	130	$63 \times 48 = 84 \times 36$	155	$76 \times 67 = 76 \times 67$	180	$88 \times 22 = 22 \times 88$
106	$51 \times 15 = 51 \times 15$	131	$64 \times 23 = 32 \times 46$	156	$77 \times 11 = 11 \times 77$	181	$88 \times 33 = 33 \times 88$
107	$52 \times 25 = 52 \times 25$	132	$64 \times 46 = 64 \times 46$	157	$77 \times 22 = 22 \times 77$	182	$88 \times 44 = 44 \times 88$
108	$53 \times 35 = 53 \times 35$	133	$64 \times 69 = 96 \times 46$	158	$77 \times 33 = 33 \times 77$	183	$88 \times 55 = 55 \times 88$
109	$54 \times 45 = 54 \times 45$	134	$65 \times 56 = 65 \times 56$	159	$77 \times 44 = 44 \times 77$	184	$88 \times 66 = 66 \times 88$
110	$55 \times 11 = 11 \times 55$	135	$66 \times 11 = 11 \times 66$	160	$77 \times 55 = 55 \times 77$	185	$88 \times 77 = 77 \times 88$
111	$55 \times 22 = 22 \times 55$	136	$66 \times 22 = 22 \times 66$	161	$77 \times 66 = 66 \times 77$	186	$88 \times 88 = 88 \times 88$
112	$55 \times 33 = 33 \times 55$	137	$66 \times 33 = 33 \times 66$	162	$77 \times 77 = 77 \times 77$	187	$88 \times 99 = 99 \times 88$
113	$55 \times 44 = 44 \times 55$	138	$66 \times 44 = 44 \times 66$	163	$77 \times 88 = 88 \times 77$	188	$89 \times 98 = 89 \times 98$
114	$55 \times 55 = 55 \times 55$	139	$66 \times 55 = 55 \times 66$	164	$77 \times 99 = 99 \times 77$	189	$91 \times 19 = 91 \times 19$

115	55 x 66 = 66 x 55	140	66 x 66 = 66 x 66	165	78 x 87 = 78 x 87	190	92 x 29 = 92 x 29
116	55 x 77 = 77 x 55	141	66 x 77 = 77 x 66	166	79 x 97 = 79 x 97	191	93 x 13 = 31 x 39
117	55 x 88 = 88 x 55	142	66 x 88 = 88 x 66	167	81 x 18 = 81 x 18	192	93 x 26 = 62 x 39
118	55 x 99 = 99 x 55	143	66 x 99 = 99 x 66	168	82 x 14 = 41 x 28	193	93 x 39 = 93 x 39
119	56 x 65 = 56 x 65	144	67 x 76 = 67 x 76	169	82 x 28 = 82 x 28	194	94 x 49 = 94 x 49
120	57 x 75 = 57 x 75	145	68 x 43 = 34 x 86	170	83 x 38 = 83 x 38	195	95 x 59 = 95 x 59
121	58 x 85 = 58 x 85	146	68 x 86 = 68 x 86	171	84 x 12 = 21 x 48	196	96 x 23 = 32 x 69
122	59 x 95 = 59 x 95	147	69 x 32 = 23 x 96	172	84 x 24 = 42 x 48	197	96 x 46 = 64 x 69
123	61 x 16 = 61 x 16	148	69 x 64 = 46 x 96	173	84 x 36 = 63 x 48	198	96 x 69 = 96 x 69
124	62 x 13 = 31 x 26	149	69 x 96 = 69 x 96	174	84 x 48 = 84 x 48	199	97 x 79 = 97 x 79
125	62 x 26 = 62 x 26	150	71 x 17 = 71 x 17	175	85 x 58 = 85 x 58	200	98 x 89 = 98 x 89

201	99 x 11 = 11 x 99
202	99 x 22 = 22 x 99
203	99 x 33 = 33 x 99
204	99 x 44 = 44 x 99
205	99 x 55 = 55 x 99
206	99 x 66 = 66 x 99
207	99 x 77 = 77 x 99
208	99 x 88 = 88 x 99
209	99 x 99 = 99 x 99

Decimal BASIC 程式

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LET i=1
FOR x=1 TO 9
FOR y=1 TO 9
FOR a= 1 TO 9
LET b=x*a/y
IF b=INT(b) AND b<10 THEN PRINT 10*x+y;"x";10*a+b;"=";10*b+a;"x";10*y+x
NEXT a
NEXT y
NEXT x
END

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