

### 三角恆等式

$$\csc(x) = \frac{1}{\sin(x)}$$

$$\sin(x) = \frac{1}{\csc(x)}$$

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\cos(x) = \frac{1}{\sec(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

$$\tan(x) = \frac{1}{\cot(x)} = \frac{\sin(x)}{\cos(x)}$$

$$\sin^2(t) + \cos^2(t) = 1$$

$$\sin(-t) = -\sin(t)$$

$$\tan^2(t) + 1 = \sec^2(t)$$

$$\cos(-t) = \cos(t)$$

$$1 + \cot^2(t) = \csc^2(t)$$

$$\tan(-t) = -\tan(t)$$

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha) \tan(\beta)}$$

## 倍角公式

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2 \sin^2(x) = 2 \cos^2(x) - 1$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

## 半角公式

$$\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos(x)}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos(x)}{1 + \cos(x)}}$$

$$\cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos(x)}{2}}$$

$$= \frac{1 - \cos(x)}{\sin(x)}$$

$$= \frac{\sin(x)}{1 + \cos(x)}$$

$$\sin^2(x) = \frac{1}{2} [1 - \cos(2x)]$$

$$\cos^2(x) = \frac{1}{2} [1 + \cos(2x)]$$

$$\tan^2(x) = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

三角形 ABC,  $\angle A = \alpha^\circ$ ,  $\angle B = \beta^\circ$ ,  $\angle C = \gamma^\circ$ ,  $\overline{AB} = c$ ,  $\overline{BC} = a$ ,  $\overline{CA} = b$

外接圓半徑是 R

$$\text{三角形 ABC 面積} = \frac{1}{2} a b \sin \gamma$$

正弦定理

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$$

餘弦定理

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$$

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta$$

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \gamma$$

### 和差化積

$$\sin(x) + \sin(y) = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) - \sin(y) = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos(x) - \cos(y) = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

### 積化和差

$$\sin(x) \cos(y) = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\cos(x) \sin(y) = \frac{1}{2} [\sin(x+y) - \sin(x-y)]$$

$$\cos(x) \cos(y) = \frac{1}{2} [\cos(x-y) + \cos(x+y)]$$

$$\sin(x) \sin(y) = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$$